Journées Arinews

Formal specifications of floating-point programs

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Disclaimer 1

This is work in progress!

Any idea or suggestion is welcomed.

Outline

Motivations

How to guarantee a critical program?

Several (complementary) approaches:

- tests (exhaustiveness?),
- pen & paper proof (safety?),
- formal proof.

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Formal methods really increase the trust the user may have in the algorithm.

However, the proof and the formal stating of the theorem are hard to read and hard to understand by non-specialists.

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How to be sure of this equality?

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You have to prove the program as it is written and make the formal statement understandable.

Existing tools

- Proof assistants -

Coq (http://coq.inria.fr)

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- few automations (automatic proofs, powerful tactics)
- Coq kernel mechanically checks each step of the proof
- in use, you successively apply tactics (lemma invocation, rewriting, simplification, hypotheses management...) to reduce the goal down to the hypotheses.

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  (It is called a decision procedure.)

Existing tools

- Formalizations of FP arithmetic -

Formalizations

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Characteristics:

- Normal floats, subnormal floats, cohorts, generic rounding, IEEE roundings, no Overflow.
- Many floats may share the same real value.
- High level formalization with mathematical definitions.

Formalization

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1.00010_2 E 4 $\mapsto$ $(100010_2, -1)_2$ $\hookrightarrow$ 17

IEEE-754 significand of the 754R real value

Existing tools

- Why -

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The tool generates proof obligations associated to the user annotations.

There is only these verification conditions to prove.

Why

Written by Jean-Christophe Filliâtre.

Java C

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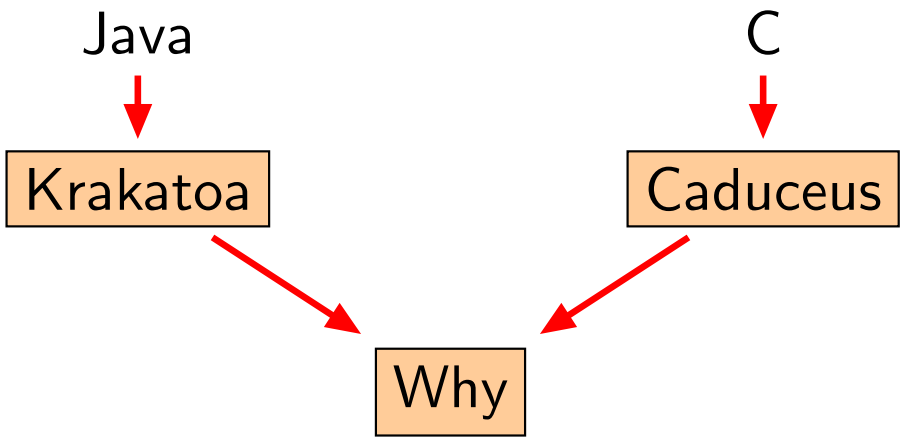

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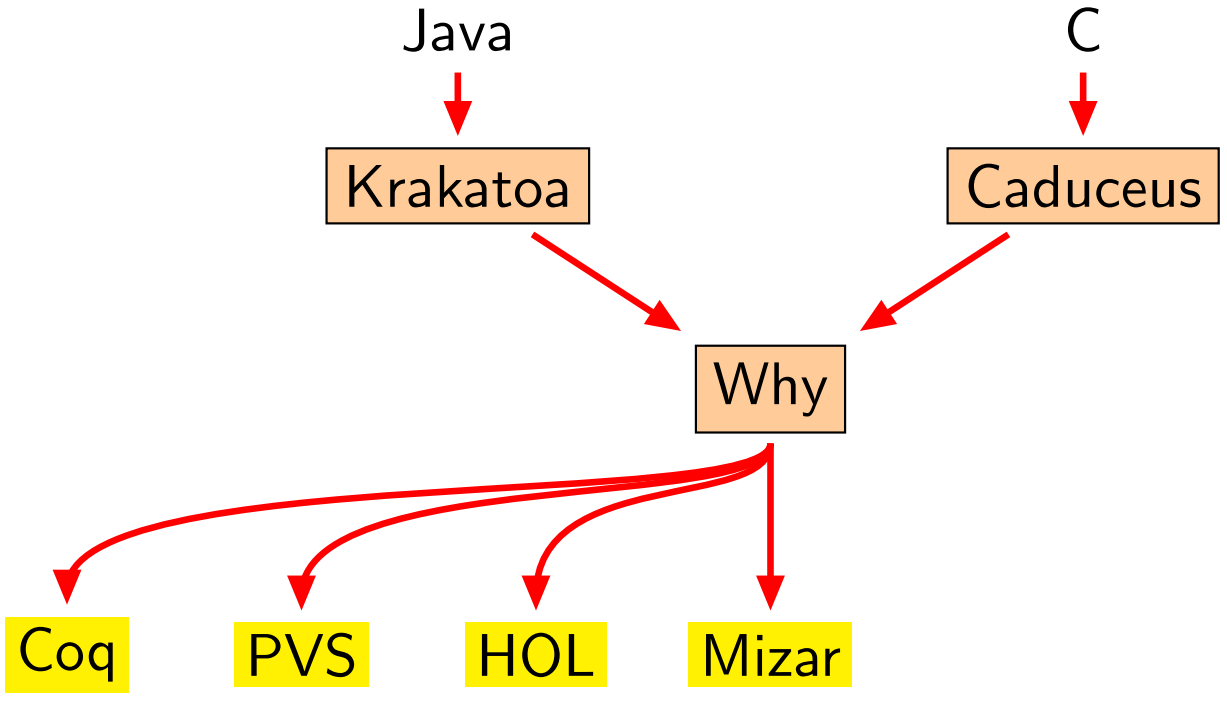

Verification conditions

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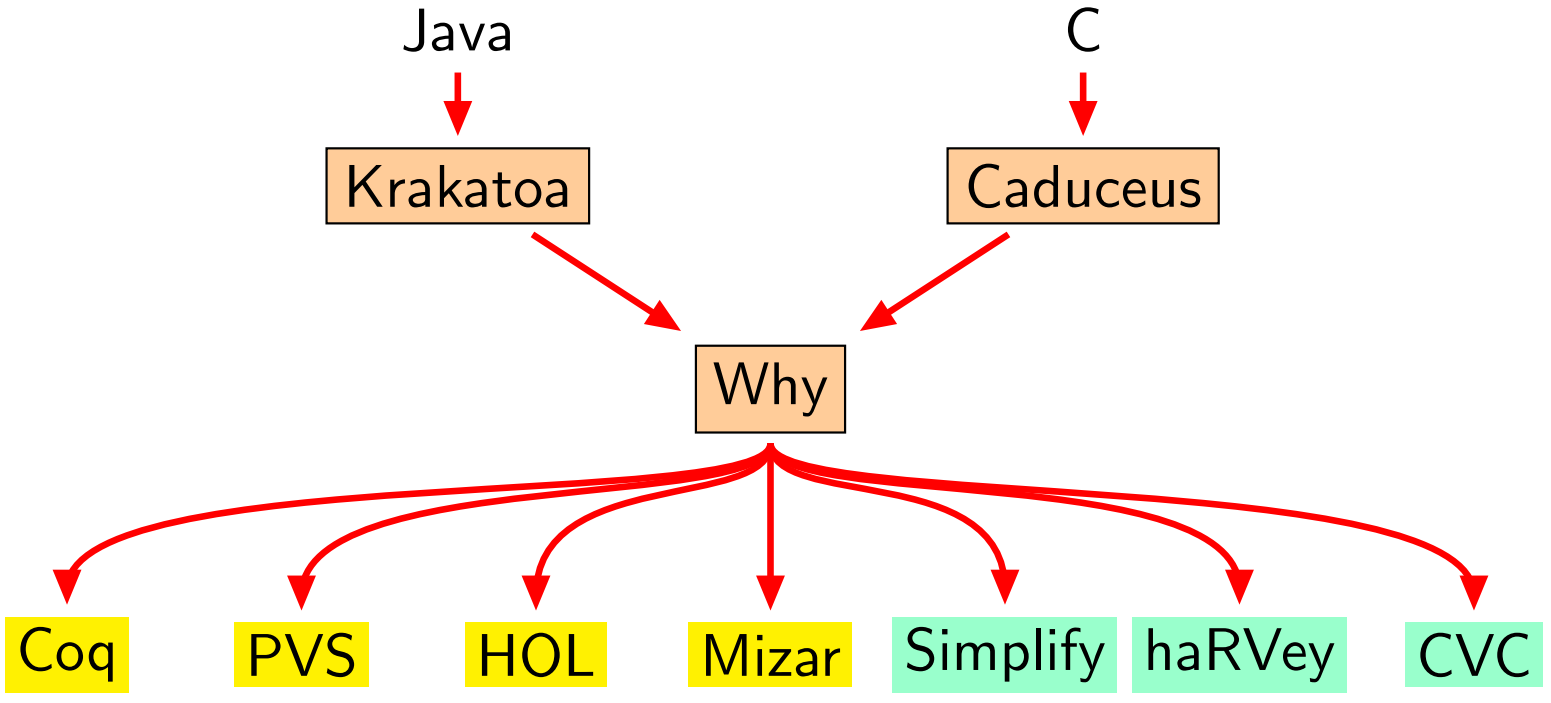

Verification conditions

Example 1

```
let swap =
  fun (t:int array)(i,j:int) ->
    { 0 <= i < array_length(t)
        and 0 <= j < array_length(t) }
    (let v = t[i] in
    begin
     t[i] := t[j];
     t[j] := v
    end)
    { exchange(t, t@, i, j) }
```

where exchange is a proof-assistant defined function.

Example 2

```
let rec quick_rec (t:int array)(l,r:int) :  unit
 { variant 1+r-l } =
  { 0 <= l and r < array_length(t) }
  (if l < r then
   let p = (partition t l r) in
   begin
    (quick_rec t l (p-1));
    (quick_rec t (p+1) r)
   end)
{ sorted_array(t, l, r) and sub_permut(l, r, t, t@) }
```

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We want to combine Why with our formalizations to be able to prove floating-point programs.

To do that, we must define the annotations we will use and the model for floating-point arithmetic.

Specification language

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Ouch...

Correct result

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The user may add assertions telling the path chosen is the same in both cases.

Disclaimer 2

The following assertions are not proved.

the following examples are not parsed by Why.

Example 1

```
/*@ requires y/2 <= x <= 2*y
  @ ensures \err = 0 */

double Sterbenz(double x, double y) {
   return x-y; }
```

Example 2

```
/*@ requires x in [0,1]
  @ ensures \result in [161,276]
  @   and \err(\result) <= 2*ulp(\result) */

double calcul(double x) {
   double a,b;
   a=2*x+3;
   b=x*x+55;
   a=a*b-4;
   return a; }
```

Example 3

```
/*@ ensures \result = exp2(53) */

  double malcolm1() {
   double A;
   A=2;   /*@ assert A=2 */

   /*@ invariant 0 <= A and A <= exp2(53)
     @ variant exp2(53)-A */
   while (A != (A+1)) {
     A*=2;
     /*@ assert A = 2*A@ */
   }
   return A; }
```

Conclusion

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$\Rightarrow$ We may use `\err(x)` in the annotations.

$\Rightarrow$ The Why float is more than an IEEE float.

$\Rightarrow$ `assert A=2` does not have a clear meaning.

$\Rightarrow$ `assert A=2` is a typing problem.

Conclusion 2

We want to provide the user with useful floating-point functions.

- `\err`,
- `ulp` (which one?),
- `exp2`.

What is missing?

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What is missing?

Note that the user may add his own functions if he defines them in the proof assistant.

Conclusion 3

What are the possible user modes?

Mode	radix	precision	
normal	2	53	
intel	2	64	for all computations?
generic-I	2	p	
generic-II	β	p	do we require $\beta = 2$ or 10?
			do we require β even?

Thank you for your attention!